LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **NOVEMBER 2024**

UMT 6501 – COMPLEX ANALYSIS

Date: 16-11-2024 Dept. No. Max. : 100 Marks
Time: 01:00 pm-04:00 pm

SECTION A - K1 (CO1)	
	Answer ALL the Questions - **(10 x 1 = 10)**
1.	**Answer the following**
a)	Show that the function $f(z)=\bar{z}$ is nowhere differential.
b)	Define harmonic function.
c)	State Maximum Moduli principle.
d)	Identify the singularity of the function $f(z)=e^{\frac{1}{z}}$.
e)	Find the residue of $f(z)=\frac{e^z}{z^2}$.
2.	**Fill in the blanks**
a)	The function $f(z)=\lvert z\rvert^2$ is differentiable only at _______________.
b)	If u and v are harmonic functions then $f(z)=u+iv$ is _________________.
c)	A Maclaurin series is a Taylor series with centre ___________________.
d)	If a function is analytic at all points inside and on a simple closed curve C , then $\oint_C f(z)dz=$______
e)	A conformal mapping preserves angle both in _______ and __________.
	SECTION A - K2 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
3.	**MCQ**
a)	Real part of $f(z)=\frac{1}{1-z}$ is _____________ (i) $\frac{1-x}{(1-x)^2+y^2}$ (ii) $\frac{1+x}{(1-x)^2+y^2}$ (iii) $\frac{1-x}{(1-y)^2+x^2}$ (iv) $\frac{1-x}{(1-x)^2-y^2}$.
b)	A function which is analytic everywhere in a complex plane is known as ___________ (i) hormonic function (ii) differential function (iii) nowhere differentiable function (iv) entire function.
c)	If the principal part of Laurent's series zero, then the Laurent's series reduces to _____________ (i) Maclaurin series (ii) Cauchy series (iii) Taylor's Series (iv) None of these.
d)	Singularities of rational functions are _____________. (i)poles (ii) essential (iii) non isolated (iv) removable.
e)	Any bilinear transformation preserves ______________. (i) cross ratio (ii) parabolic (iii) hyperbolic (iv) None of these.
4.	**True or False**
a)	If Cauchy Riemann equations are satisfied at z_0, then $f(z)$ is differentiable at z_0.
b)	A domain that is not simply connected is said to be multiply connected.
c)	$u(x,y)$is a harmonic conjugate of $v(x,y)$ if $f(z)=u(x,y)+iv(x,y)$ is an analytic function.

d)	A bounded entire function in the complex plane is constant.
e)	Point at which $f'(z)=0$ is called as critical point of the transformation.

SECTION B - K3 (CO2)	
Answer any TWO of the following	**(2 x 10 = 20)**
5.	Verify C-R equations for $f(z)=\begin{cases}\frac{xy^2}{x^2+y^2}, z\neq 0\\ 0, z=0\end{cases}$. Can you say the this function is differentiable at $z=0$?
6.	Find the constant *a* so that $u(x,y)=ax^2-y^2+xy$ is harmonic. Find an analytic function $f(z)$ for which *u* is real. Also find its harmonic conjugate.
7.	Expand $f(z)=\frac{z-1}{z+1}$ as a Taylor's series (i) about the point *z* =0 (ii) about the point *z* =1. Determine the region of convergence in each case.
8.	Show that the circle $\lvert z-3\rvert=5$ is mapped into the circle $\left\lvert w+\frac{3}{16}\right\rvert=\frac{5}{16}$ by the transformation $w=\frac{1}{z}$.
SECTION C – K4 (CO3)	
Answer any TWO of the following	**(2 x 10 = 20)**
9.	Show that $f(z)=\sqrt{r}¿$ where $r>0$ & $0<\theta<2\pi$ is differentiable and find $f'(z)$.
10.	Using Cauchy's integral formula, prove that $\int_C \frac{z\,dz}{z^2-1}=2\pi i$, C is $\lvert z\rvert=2$.
11.	Determine the residues of $\frac{z+1}{z^2-2z}$ at its poles.
12.	Find the singularities for the following functions and classify them: (i) $\frac{\sin z}{z}$ (ii) $\left(\frac{z^2-2z+3}{z-2}\right)$.
SECTION D – K5 (CO4)	
Answer any ONE of the following	**(1 x 20 = 20)**
13.	State a necessary and sufficient condition for a complex valued function to be differentiable at a point *z* and defend them.
14.	Expand $f(z)=\frac{z}{(z-1)(2-z)}$ in a Laurent's series valid for (i) $\lvert z\rvert<1$ (ii) $1<\lceil z \rceil<2$ (iii) $\lvert z-1\rvert>1$.
SECTION E – K6 (CO5)	
Answer any ONE of the following	**(1 x 20 = 20)**
15.	Device the proof of Cauchy's integral formula
16.	Defend Cauchy's Residue theorem and hence evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$.

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